Answer on Question #78866 – Math – Calculus

Question

Find the length of the curve y=ln {(e^x-1)/(e^x+1)} from x=1 to x=2.

Solution

$$L = \int_1^2 \sqrt{1 + \left(\left(\ln\frac{e^x - 1}{e^x + 1}\right)'\right)^2}\, dx = \int_1^2 \sqrt{1 + \left(\frac{2e^x}{e^{2x} - 1}\right)^2}\, dx = \int_1^2 \frac{e^{2x} + 1}{e^{2x} - 1} dx$$

$$= \int_1^2 \frac{e^{2x} - 1}{e^{2x} - 1} dx$$

$$+ \ 2\int_1^2 \frac{1}{e^{2x} - 1} dx = \int_1^2 dx + \ 2\int_1^2 \frac{e^{2x}}{2e^{2x}(e^{2x} - 1)} d(2x) =$$

$$= \left|\frac{1}{2}\int_{e^2}^{e^4} \left(\frac{1}{t - 1} - \frac{1}{t}\right) dt = |\ t = e^{2x}| = \frac{1}{2}(\ln(t - 1) - \ln(t))\right|$$

$$= x|_1^2 + \ 2 \cdot \left(\frac{1}{2}(\ln(t - 1) - \ln(t))\right)\Bigg|_{e^2}^{e^4} = 1 \ + \ 0.12692801..$$

$$\approx 1.1269.$$