

Answer on Question #78838 – Math – Statistics and Probability

Question

The intelligence quotients (IQs) of 16 students from one area of a city showed a mean of 107 and a standard deviation of 10, while IQ of 14 students from another area of the city showed a mean of 112 and a standard deviation of 8. Is there a significant difference between IQs of the two groups at significance level of

(a) 0.01

and

(b) 0.05?

Solution

$$n_1 = 16, \bar{x}_1 = 107, S_1 = 10$$

$$n_2 = 14, \bar{x}_2 = 112, S_2 = 8$$

Since $\frac{1}{2} \leq \frac{S_1}{S_2} \leq 2$, we have equal variances.

To test the hypothesis of a significant difference between the mean we can apply t-test using of the total variance.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (m_1 - m_2)}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \text{ where } S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} \text{ is the total variance.}$$

$$S_p^2 = \frac{15 \cdot 100 + 13 \cdot 64}{15 + 13} = \frac{2332}{28} = 83.286$$

$$S_p = \sqrt{S_p^2} = 9.126$$

a.

$$(\bar{x}_1 - \bar{x}_2) - t(0.01, n_1 + n_2 - 2) \cdot \sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \leq (m_1 - m_2) \leq (\bar{x}_1 - \bar{x}_2) + t(0.01, n_1 + n_2 - 2) \cdot \sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$-5 - t(0.01, 28) * 9.126 * 0.366 \leq (m_1 - m_2) \leq -5 + t(0.01, 28) * 9.126 * 0.336$$

$$-5 - 2.467 * 3.341 \leq (m_1 - m_2) \leq -5 + 2.467 * 3.341$$

$$-5 - 8.241 \leq (m_1 - m_2) \leq -5 + 8.241$$

$$[-13.241; 3.241]$$

Answer: Since the 99% confidence interval contains 0, there is no significant difference between IQs of the two groups at significance level of 0.01.

b.

$$\begin{aligned}(\bar{x}_1 - \bar{x}_2) - t(0.05, n_1 + n_2 - 2) * \sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)} &\leq (m_1 - m_2) \leq (\bar{x}_1 - \bar{x}_2) + \\ + t(0.05, n_1 + n_2 - 2) * \sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)} & \\ -5 - t(0.05, 28) * 9.126 * 0.366 &\leq (m_1 - m_2) \leq -5 + t(0.05, 28) * 9.126 * 0.336 \\ -5 - 1.701 * 3.341 &\leq (m_1 - m_2) \leq -5 + 1.701 * 3.341 \\ -5 - 5.683 &\leq (m_1 - m_2) \leq -5 + 5.683 \\ [-10.683; 0.683] &\end{aligned}$$

Answer: Since the 95% confidence interval contains 0, there is no significant difference between IQs of the two groups at significance level of 0.05.