Answer on Question #78800 – Math – Differential Equations

Charging characteristics for a series capacitive circuit is

$$V_C = V\left(1 - e^{-\frac{t}{T}}\right),$$

where $T = CR,$ time is constant;
Capacitor, $C = 100\ nF$;
Resistor, $R = 47\ k\Omega$;
Supply voltage, $V = 5\ Volts$.

Question

1. Determine the value of t when $V_C = 4.15\ Volts$.

Solution

$$V_C = V\left(1 - e^{-\frac{t}{T}}\right)$$

Solve for t

$$1 - e^{-\frac{t}{T}} = \frac{V_C}{V}$$

$$e^{-\frac{t}{T}} = 1 - \frac{V_C}{V}$$

$$-\frac{t}{T} = \ln\left(1 - \frac{V_C}{V}\right)$$

$$t = -T\ln\left(1 - \frac{V_C}{V}\right)$$

$$t = -RC\ln\left(1 - \frac{V_C}{V}\right)$$

Substitute

$$t = -(47 \times 10^3\ \Omega)(100 \times 10^{-9} F)\ln\left(1 - \frac{4.15\ Volts}{5\ Volts}\right)$$

$$t = 0.00832820\ s \approx 8.328\ \times 10^{-3}\ s = 8.328\ \ ms$$

Answer: $t = 8.328\ \ ms$

Question

2. Differentiate the charging equation and find the rate of change of voltage at $6\ ms$.

Solution

$$V_C = V\left(1 - e^{-\frac{t}{T}}\right)$$

Differentiate both sides with respect to t

$$\frac{d}{dt}(V_C) = \frac{d}{dt}\left(V\left(1 - e^{-\frac{t}{T}}\right)\right)$$

$$rate\ of\ change\ of\ voltage = \frac{dV_C}{dt} = V\left(\frac{1}{T}\right)e^{-\frac{t}{T}} = \frac{V}{RC}e^{-\frac{t}{RC}}$$

Capacitor, $C = 100\ nF$
Resistor, $R = 47\ k\Omega$
Supply voltage, $V = 5\ Volts$
$t = 6\ ms$

$$rate\ of\ change\ of\ voltage = \frac{dV_C}{dt} =$$

$$= \frac{5\ Volts}{(47 \times 10^3\ \Omega)(100 \times 10^{-9} F\ )} e^{-\frac{6\times 10^{-3} s}{(47\times 10^3\ \Omega)(100\times 10^{-9} F\ )}} \approx 296.793\ Volts/s$$

Answer: $rate\ of\ change\ of\ voltage = \frac{dV_C}{dt} = V\left(\frac{1}{T}\right)e^{-\frac{t}{T}} = \frac{V}{RC}e^{-\frac{t}{RC}}$

$$rate\ of\ change\ of\ voltage|_{t=6ms} = \frac{dV_C}{dt}|_{t=6ms} = 296.793\ Volts/s$$