Example. Find the unique solution of the initial value problem:

$$y' - ay = 0, \;\; y(x_0) = y_0.$$

Solution: Rewriting $y' = ay$ we get

$$\frac{y'}{y} = a \quad \Rightarrow \quad \ln(|y|)' = a \quad \Rightarrow \quad \ln(|y|) = ax + c_0.$$

We now compute exponentials on both sides, to get

$$y(x) = \pm e^{ax+c_0} = \pm e^{ax} e^{c_0}, \;\; \text{denote } c = \pm e^{c_0}, \;\; \text{then } y(x) = ce^{ax},$$

where c is an arbitrary constant. The initial condition determines c,

$$y_0 = y(x_0) = ce^{ax_0} \quad \Rightarrow \quad c = y_0 e^{-ax_0}.$$

Then, the unique solution to the initial value problem above is

$$y(x) = y_0 e^{-ax_0} e^{ax} = y_0 e^{a(x-x_0)}.$$