Answer on Question #78593 – Math – Analytic Geometry

Question

Trace the surface

$$\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{4}=1$$

Also describe its sections by the planes $x=\pm 2$, algebraically and geometrically.

Solution

One-sheeted hyperboloid.

To determine the xy −trace, set $z=0$

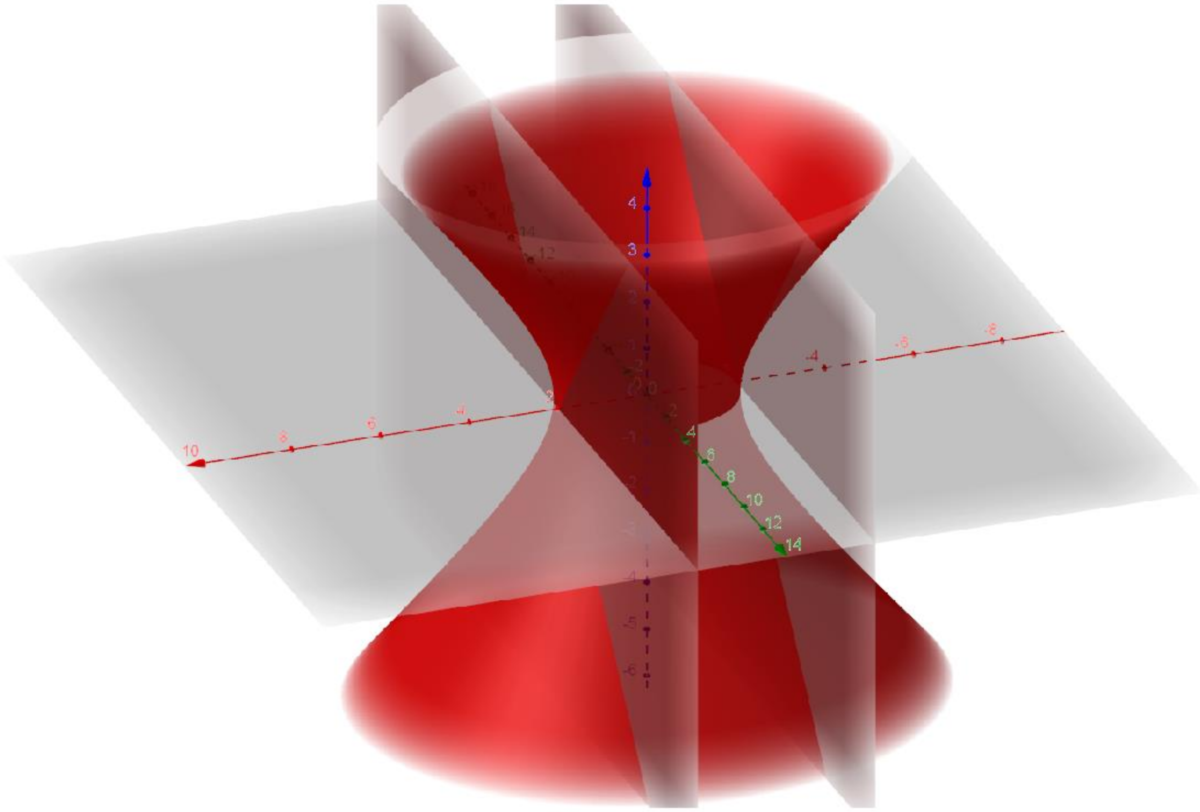
$$\frac{x^2}{4}+\frac{y^2}{9}=1 \qquad \text{the } xy-\text{trace is the ellipse}$$

To determine the xz −trace, set $y=0$

$$\frac{x^2}{4}-\frac{z^2}{4}=1 \qquad \text{the } xz-\text{trace is the hyperbola}$$

To determine the yz −trace, set $x=0$

$$\frac{y^2}{9}-\frac{z^2}{4}=1 \qquad \text{the } yz-\text{trace is the hyperbola}$$



The section of the hyperboloid by the plane $x=-2$

$$\frac{(-2)^2}{4}+\frac{y^2}{9}-\frac{z^2}{4}=1$$

$$\frac{y^2}{9}-\frac{z^2}{4}=0$$

$$\left(\frac{y}{3}-\frac{z}{2}\right)\left(\frac{y}{3}+\frac{z}{2}\right)=0$$

Two lines: $y=-\frac{3}{2}z$ and $y=\frac{3}{2}z$.

The section of the hyperboloid by the plane $x=2$

$$\frac{2^2}{4}+\frac{y^2}{9}-\frac{z^2}{4}=1$$

$$\frac{y^2}{9}-\frac{z^2}{4}=0$$

$$\left(\frac{y}{3}-\frac{z}{2}\right)\left(\frac{y}{3}+\frac{z}{2}\right)=0$$

Two lines: $y=-\frac{3}{2}z$ and $y=\frac{3}{2}z$.