Answer on Question #78514 – Math – Analytic Geometry

Question

2x + 3y = 7z represents a line in three-dimensional space.

Is the statement true? Give reason for your answer, either with a short proof or a counterexample.

Solution

The general form of the equation of the plane has the form

$$ax + by + cz + d = 0,$$

where the vector $\vec{n} = (a, b, c)$ is the normal to plane, and d gives the point the plane passing through. As we can see our equation

$$2x + 3y - 7z = 0$$

has exactly the same form, so that is not a line, but a plane. A line can be given as an intersection of two planes, so with this equation we need at least one extra, with independent coefficients.

Let's show that this equation gives us an infinite number of lines. The parametric equations of a line are (with two points (x_0, y_0, z_0) and (x_1, y_1, z_1)):

$$\begin{cases} x = x_0 + (x_1 - x_0)t; \\ y = y_0 + (y_1 - y_0)t; \\ z = z_0 + (z_1 - z_0)t. \end{cases}$$

We take first point $(x_0, y_0, z_0) = (0,0,0)$, that satisfying our equation.

$$\begin{cases} x = x_1 t; \\ y = y_1 t; \\ z = z_1 t. \end{cases}$$

Now we substitute these values into equation:

$$2x_1 t + 3y_1 t - 7z_1 t = 0;$$

$$2x_! + 3y_1 - 7z_1 = 0.$$

One can see that the set of numbers (x_1, y_1, z_1), that means the set of different second points and the set of different lines can be infinite.

Therefore, this statement is not true.

Answer: this statement is not true.