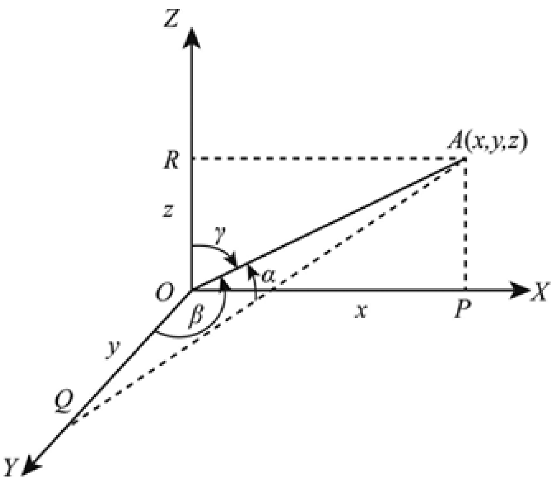
Answer on Question #78513 – Math – Analytic Geometry

Question

$1/\sqrt{2}$, $1/\sqrt{3}$, $1/\sqrt{5}$ form the direction cosines of a line.

Is the statement true? Give reason for your answer, either with a short proof or a counterexample.

Solution



The direction cosine of a line is defined as the cosine of the angles between the positive directed lines and the coordinate axes. If α, β and γ are the three angles between the directed line segment and the coordinate axes, then these three angles are considered as direction angles. The cosine of these directed angles, cos α, cos β, and cos γ are termed as direction cosines of the line with general notation l, m, and n, respectively. That is,

$$l = \cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}};$$

$$m = \cos\beta = \frac{y}{\sqrt{x^2+y^2+z^2}};$$

$$n = \cos\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}}.$$

The relation between the direction cosines is as follows:

$$l^2+m^2+n^2 = \frac{x^2}{x^2+y^2+z^2}+\frac{y^2}{x^2+y^2+z^2}+\frac{z^2}{x^2+y^2+z^2} = 1.$$

So we have to check the following conditions:

a) $l, m, n \le 1$ (because cosine is always ≤ 1);
b) $l^2+m^2+n^2 = 1$.

We can see that condition a) is satisfied:

$$\frac{1}{\sqrt{2}} < 1;\ \frac{1}{\sqrt{3}} < 1;\ \frac{1}{\sqrt{5}} < 1.$$

But condition b) is not satisfied:

$$\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{5}}\right)^2 = \frac{1}{2}+\frac{1}{3}+\frac{1}{5} = \frac{31}{30} \ne 1.$$

Therefore, this statement is not true.

Answer: this statement is not true.