Answer on Question #78433 – Math – Complex Analysis

Question

Obtain the polar and exponential representations of z_1, z_2, and $z_1 z_2$ where $z_1 = 1/2 - 2i$, and $z_2 = 3 + i$.

Solution

$$z_1 z_2 = \left(\frac{1}{2} - 2i\right)(3 + i) = \frac{7}{2} - \frac{11}{2} i$$

Exponential form:

z_1:

$$|z_1| = \sqrt{0.5^2 + (-2)^2} = \frac{\sqrt{17}}{2}, \ \ \operatorname{Arg} z_1 = \tan^{-1}\left(\frac{-2}{1/2}\right) = \tan^{-1}(-4)$$

$$z_1 = |z_1| e^{i \operatorname{Arg} z_1} = \frac{\sqrt{17}}{2} e^{i \tan^{-1}(-4)}$$

z_2:

$$|z_2| = \sqrt{3^2 + 1^2} = \sqrt{10}, \ \ \operatorname{Arg} z_2 = \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}(1/3)$$

$$z_2 = |z_2| e^{i \operatorname{Arg} z_2} = \sqrt{10} e^{i \tan^{-1}(1/3)}$$

$z_1 z_2$:

$$|z_1 z_2| = |z_1||z_2| = \frac{\sqrt{170}}{2}, \ \ \operatorname{Arg} z_1 z_2 = \tan^{-1}\left(\frac{-11/2}{7/2}\right) = \tan^{-1}(-11/7)$$

$$z_1 z_2 = |z_1 z_2| e^{i \operatorname{Arg} z_1 z_2} = \frac{\sqrt{170}}{2} e^{i \tan^{-1}(-11/7)}$$

Polar form:

Since $e^{i\theta} = \cos\theta + i \sin\theta$, we obtain

$$z_1 = \frac{\sqrt{17}}{2}(\cos(\tan^{-1}(-4)) + i \sin(\tan^{-1}(-4)))$$

$$z_2 = \sqrt{10}(\cos(\tan^{-1}(1/3)) + i \sin(\tan^{-1}(1/3)))$$

$$z_1 z_2 = \frac{\sqrt{170}}{2}(\cos(\tan^{-1}(-11/7)) + i \sin(\tan^{-1}(-11/7)))$$

Answer:

$$z_1 = \frac{\sqrt{17}}{2} e^{i \tan^{-1}(-4)} = \frac{\sqrt{17}}{2} (\cos(\tan^{-1}(-4)) + i \sin(\tan^{-1}(-4))),$$

$$z_2 = \sqrt{10} e^{i \tan^{-1}(1/3)} = \sqrt{10} (\cos(\tan^{-1}(1/3)) + i \sin(\tan^{-1}(1/3))),$$

$$z_1 z_2 = \frac{\sqrt{170}}{2} e^{i \tan^{-1}(-11/7)} = \frac{\sqrt{170}}{2} (\cos(\tan^{-1}(-11/7)) + i \sin(\tan^{-1}(-11/7))).$$