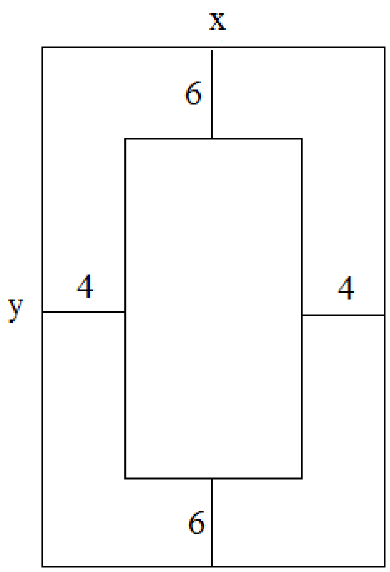
Answer on Question #77049 – Math – Calculus

Question

The top and bottom margins of a poster are each 6 cm, and the side margins are each 4 cm. If the area of the printed material on the poster (that is, the area between the margins) is fixed at 384 cm^2, find the dimensions of the poster with the smallest total area.

Solution



Let x be the length of the poster in cm, y be the width of the poster in cm. Then the area of the poster

$$A = xy \qquad (*)$$

The area of the printed material on the poster

$$A_{pr} = \big(x - 2(4)\big)\big(y - 2(6)\big) = 384$$

Solve the equation for y

$$y - 12 = \frac{384}{x-8}$$

$$y = 12 + \frac{384}{x-8}$$

Substitute into $(*)$

$$A = x\left(12 + \frac{384}{x-8}\right)$$

We see that the area of the poster is the function on x

$$A = A(x) = 12x + \frac{384x}{x-8}, x > 8$$

Find the first derivative with respect to x

$$\frac{dA}{dx} = \frac{d}{dx}\left(12x + \frac{384x}{x-8}\right) = 12 + 384\frac{(x)'(x-8) - (x-8)'(x)}{(x-8)^2} =$$

$$= 12 + 384\frac{x - 8 - x}{(x - 8)^2} = 12 - \frac{3072}{(x - 8)^2}$$

Find the critical number(s)

$\frac{dA}{dx} = 0 => 12 - \frac{3072}{(x - 8)^2} = 0\ , x > 8$

$(x - 8)^2 = 256$

Since $x > 8$, we take

$x - 8 = 16$

$x = 24$

$A(24) = 12(24) + \frac{384(24)}{24 - 8} = 864$

$If\ 8 < x < 24, \frac{dA}{dx} < 0, A(x)\ decreases$

$If\ x > 24, \frac{dA}{dx} > 0, A(x)\ increases$

The function $A(x)$ has a local minimum at $x = 24$.

Since the function $A(x)$ has only one extrema on $(8, \infty)$, then the function $A(x)$ has the absolute minimum with value of 864 at $x = 24$ on $(8, \infty)$.

Find y, when $x = 24$

$y = 12 + \frac{384}{24 - 8} = 36$

Answer: the given poster has the minimal area with value of 864 cm^2 when its length is 24 cm and its width is 36 cm.