Answer on Question #76991 – Math – Calculus

Question

Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola

$$y = 12 - x^2$$

Solution

See the figure 1 below:

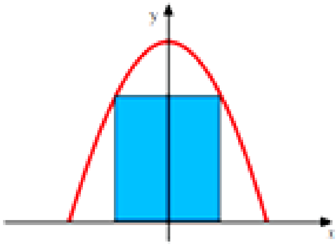

Figure 1

The area of the inscribed rectangle equals

$$A(x) = 2x\,(12 - x^2) = 24x - 2x^3 \; for \;\; 0 \le x \le 2\sqrt{3}.$$

Calculate the derivative of the area function, and set it equal to zero.

$$A'(x) = 24 - 6x^2 = 0$$
$$x^2 = 4$$
$$x = 2$$

As $A'(x) > 0$ for $0 \le x < 2$, and $A'(x) < 0$ for $2 < x \le 2\sqrt{3}$, then $x = 2$ is the maximum point of $A(x)$ and the rectangle has the largest area of 32.

Length of the rectangle is $2x = 2 \cdot 2 = 4$; the height of the rectangle is $12 - x^2 = 12 - 2^2 = 8$.

Answer: The rectangle has the length 4 and the height of the rectangle is 8.

For more details see an example in [1].

References:

1. http://www.math.tamu.edu/~stecher/151/Sp00/final.pdf