Answer on Question #76989 – Math – Calculus

Question

A particle starts from rest (that is with initial velocity zero) at the point x = 10 and moves along the x-axis with acceleration function a(t) = 12t. Find the resulting position function x(t).

Solution

We have initial conditions: $x(t=0)=x_0=10;\ \ v(t=0)=v_0=0;\ \ a(t)=12t.$

Definition of acceleration function:

$$a(t)=\ddot{x}(t)=\frac{d^2x}{dt^2}=\frac{dv}{dt}.$$

Definition of velocity function:

$$v(t)=\dot{x}(t)=\frac{dx}{dt}.$$

Let's find the velocity of a particle:

$$v(t)=\int a(t)dt=\int 12tdt=12\frac{t^2}{2}+C_1=6t^2+C_1.$$

Constant of integration C_1 can be found from initial conditions:

$$v(t=0)=6\cdot 0+C_1=v_0=0\ \ \Rightarrow\ \ C_1=0.$$

So we have

$$v(t)=6t^2.$$

Let's find the position function:

$$x(t)=\int v(t)dt=\int 6t^2dt=6\frac{t^3}{3}+C_2=2t^3+C_2.$$

Constant of integration C_2 we find from initial conditions:

$$x(t=0)=2\cdot 0+C_2=x_0=10\ \ \Rightarrow\ \ C_2=10.$$

So the equation for position function:

$$x(t)=2t^3+10.$$