Answer on Question #76845 – Math – Statistics and Probability

The management of a grocery store has kept a record of bad checks received per day for a period of 300 days.

Number of Bad Checks Received	Number of Days
0	12
1	15
2	35
3	55
4	63
5	35
6	38
7	32
8	15

Question

a) Develop a probability distribution for the above data.

Solution

Number of Bad Checks Received	Probability $p(x)$
0	$12/300 = 0.04$
1	$15/300 = 0.05$
2	$35/300 = 0.117$
3	$55/300 = 0.183$
4	$63/300 = 0.21$
5	$35/300 = 0.117$
6	$38/300 = 0.126$
7	$32/300 = 0.107$
8	$15/300 = 0.05$

Question

b) Is the probability distribution that you found in Part "a" a proper probability distribution? Explain.

Solution

This is a proper probability distribution since:

$$p(x_i) \geq 0 \; for \; all \; x$$

$$\sum_{i=0}^{8} p(x_i) = 1$$

Question

c) Determine the cumulative probability distribution, $F(x)$.

Solution

$$F(x) = \sum_{i:\, x_i \le x} p_i$$

$$F(0) = p(x = 0) = 0.04$$

$$F(1) = p(x \le 1) = 0.04 + 0.05 = 0.09$$

$$F(2) = p(x \le 2) = 0.04 + 0.05 + 0.117 = 0.27$$

$$F(3) = p(x \le 3) = 0.04 + 0.05 + 0.117 + 0.183 = 0.39$$

$$F(4) = p(x \le 4) = 0.39 + 0.21 = 0.6$$

$$F(5) = p(x \le 5) = 0.6 + 0.117 = 0.717$$

$$F(6) = p(x \le 6) = 0.717 + 0.126 = 0.843$$

$$F(7) = p(x \le 7) = 0.843 + 0.107 = 0.95$$

$$F(8) = p(x \le 8) = 1$$

Question

d) What is the probability that in a given day the store receives four or less bad checks?

Solution

$$p(x \le 4) = F(4) = 0.21 + 0.183 + 0.117 + 0.05 + 0.04 = 0.6$$

Question

e) What is the probability that in a given day the store receives more than two bad checks?

Solution

$$p(x > 2) = 1 - p(x \leq 2) = 1 - 0.04 - 0.05 - 0.117 = 0.793$$

Question

f) What is the expected value of the number of checks received?

Solution

$$\mu = \sum_{i=0}^{8} p_i x_i = 0.05 + 0.117 \cdot 2 + 0.183 \cdot 3 + 0.21 \cdot 4 + 0.117 \cdot 5 + 0.126 \cdot 6 + 0.107 \cdot 7 +$$
$$+0.05 \cdot 8 = 4.163$$

Question

g) Compute the variance of the number of checks received.

Solution

$$var(x) = \sum_{i=0}^{8} p_i x_i^2 - \mu^2 = 0.05 + 0.117 \cdot 4 + 0.183 \cdot 9 + 0.21 \cdot 16 + 0.117 \cdot 25 +$$
$$+0.126 \cdot 36 + 0.107 \cdot 49 + 0.05 \cdot 64 - 4.163^2 = 4.098$$

Question

h) Compute the standard deviation of number of checks received.

Solution

$$\sigma = \sqrt{var(x)} = \sqrt{4.098} = 2.024$$