Answer on Question #76748 – Math – Calculus

Question

By using the formula, $f' = \lim_{z\to x}\frac{f(z)-f(x)}{z-x}$, find the derivatives of the following functions.

a) $f(x) = \frac{1}{x+2}$

b) $f(x) = \frac{1}{(x-1)^2}$

c) $g(x) = \frac{x}{x-1}$

d) $g(x) = 1+\sqrt{x}$

Solution

a) $f(x) = \frac{1}{x+2}$

$$f'(x) = \lim_{z\to x}\frac{f(z)-f(x)}{z-x} = \lim_{z\to x}\frac{1}{z-x}\left\{\frac{1}{z+2}-\frac{1}{x+2}\right\}$$

$$= \lim_{z\to x}\frac{(x+2)-(z+2)}{(z-x)(z+2)(x+2)} = \lim_{z\to x}\frac{-(z-x)}{(z-x)(z+2)(x+2)}$$

$$= \lim_{z\to x}\frac{-1}{(z+2)(x+2)} = -\frac{1}{(x+2)^2}$$

b) $f(x) = \frac{1}{(x-1)^2}$

$$f'(x) = \lim_{z\to x}\frac{f(z)-f(x)}{z-x} = \lim_{z\to x}\frac{1}{z-x}\left\{\frac{1}{(z-1)^2}-\frac{1}{(x-1)^2}\right\}$$

$$= \lim_{z\to x}\frac{(x-1)^2-(z-1)^2}{(z-x)(z-1)^2(x-1)^2} = \lim_{z\to x}\frac{(x-1)^2-((z-x)+(x-1))^2}{(z-x)(z-1)^2(x-1)^2}$$

$$= \lim_{z\to x}\frac{(x-1)^2-(x-1)^2-2(\mathrm{x}-1)(z-x)-(z-x)^2}{(z-x)(z-1)^2(x-1)^2}$$

$$= \lim_{z\to x}\frac{-2(\mathrm{x}-1)(z-x)-(z-x)^2}{(z-x)(x+\Delta x-1)^2(x-1)^2} = \lim_{z\to x}\frac{-2(\mathrm{x}-1)-(z-x)}{(z-1)^2(x-1)^2}$$

$$= \lim_{z\to x}\frac{2-z-x}{(z-1)^2(x-1)^2} = -\frac{2}{(x-1)^3}$$

c) $g(x) = \frac{x}{x-1}$

$$g'(x) = \lim_{z \to x} \frac{g(z) - g(x)}{z - x} = \lim_{z \to x} \frac{1}{\Delta x} \left\{ \frac{z}{z-1} - \frac{x}{x-1} \right\}$$

$$= \lim_{z \to x} \frac{z(x-1) - x(z-1)}{(z-x)(z-1)(x-1)} = \lim_{z \to x} \frac{-(z-x)}{(z-x)(z-1)(x-1)}$$

$$= \lim_{z \to x} \frac{-1}{(z-1)(x-1)} = -\frac{1}{(x-1)^2}$$

d) $g(x) = 1 + \sqrt{x}$

$$g'(x) = \lim_{z \to x} \frac{g(z) - g(x)}{z - x} = \lim_{z \to x} \frac{(1 + \sqrt{z}) - (1 + \sqrt{x})}{z - x}$$

$$= \lim_{z \to x} \frac{\sqrt{z} - \sqrt{x}}{z - x} = \lim_{z \to x} \frac{(\sqrt{z} - \sqrt{x})}{z - x} \frac{(\sqrt{z} + \sqrt{x})}{(\sqrt{z} + \sqrt{x})}$$

$$= \lim_{z \to x} \frac{(\sqrt{z})^2 - (\sqrt{x})^2}{(z-x)(\sqrt{z} + \sqrt{x})} = \lim_{z \to x} \frac{z - x}{(z-x)(\sqrt{z} + \sqrt{x})}$$

$$= \lim_{z \to x} \frac{1}{(\sqrt{z} + \sqrt{x})} = \frac{1}{2\sqrt{x}}$$

Answer: the derivatives of the functions:

a) $f'(x) = -\frac{1}{(x+2)^2}$ for $f(x) = \frac{1}{x+2}$

b) $f'(x) = -\frac{2}{(x-1)^3}$ for $f(x) = \frac{1}{(x-1)^2}$

c) $g'(x) = -\frac{1}{(x-1)^2}$ for $g(x) = \frac{x}{x-1}$

d) $g'(x) = \frac{1}{2\sqrt{x}}$ for $g(x) = 1 + \sqrt{x}$