Answer on Question #76676, Math / Calculus

1. By using the formula,

$$f'(x)=\lim_{z\to x}\frac{f(z)-f(x)}{z-x},$$

find the derivatives of the following functions.

$a)\quad f(x)=\frac{1}{x+2}$

Solution

$$f'(x)=\lim_{z\to x}\frac{f(z)-f(x)}{z-x}=\lim_{z\to x}\frac{\frac{1}{z+2}-\frac{1}{x+2}}{z-x}=\lim_{z\to x}\frac{x+2-z-2}{(z-x)(z+2)(x+2)}=$$

$$=\lim_{z\to x}\frac{-(z-x)}{(z-x)(z+2)(x+2)}=\lim_{z\to x}\frac{-1}{(z+2)(x+2)}=-\frac{1}{(x+2)^2}$$

$b)\quad f(x)=\frac{1}{(x+2)^2}$

Solution

$$f'(x)=\lim_{z\to x}\frac{f(z)-f(x)}{z-x}=\lim_{z\to x}\frac{\frac{1}{(z+2)^2}-\frac{1}{(x+2)^2}}{z-x}=$$

$$=\lim_{z\to x}\frac{(x+2+z+2)(x+2-z-2)}{(z-x)(z+2)^2(x+2)^2}=\lim_{z\to x}\frac{-(x+z+4)(z-x)}{(z-x)(z+2)^2(x+2)^2}=$$

$$=\lim_{z\to x}\frac{-(x+z+4)}{(z+2)^2(x+2)^2}=-\frac{x+x+4}{(x+2)^4}=-\frac{2}{(x+2)^3}$$

$c)\quad g(x)=\frac{x}{x-1}$

Solution

$$g'(x)=\lim_{z\to x}\frac{f(z)-f(x)}{z-x}=\lim_{z\to x}\frac{\frac{z}{z-1}-\frac{x}{x-1}}{z-x}=\lim_{z\to x}\frac{zx-z-zx+x}{(z-x)(z-1)(x-1)}=$$

$$=\lim_{z\to x}\frac{-(z-x)}{(z-x)(z-1)(x-1)}=\lim_{z\to x}\frac{-1}{(z-1)(x-1)}=-\frac{1}{(x-1)^2}$$

$d)\quad g(x)=1+\sqrt{x}$

Solution

$$g'(x)=\lim_{z\to x}\frac{f(z)-f(x)}{z-x}=\lim_{z\to x}\frac{1+\sqrt{z}-\left(1+\sqrt{x}\right)}{z-x}=\lim_{z\to x}\frac{\sqrt{z}-\sqrt{x}}{z-x}=$$

$$=\lim_{z\to x}\left(\frac{\sqrt{z}-\sqrt{x}}{z-x}\cdot\frac{\sqrt{z}+\sqrt{x}}{\sqrt{z}+\sqrt{x}}\right)=\lim_{z\to x}\frac{z-x}{(z-x)(\sqrt{z}+\sqrt{x})}=\lim_{z\to x}\frac{1}{\sqrt{z}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$