Question

What is the maximum number of relative extrema contained in the graph of this function?

$$f(x) = 3x^3 - x^2 + 4x - 2$$

Solution

Since $f(x)$ is a polynomial function, the number of relative extrema is, at most, one less than the degree of the polynomial.

So, for this particular function, the number of relative extrema is 2 or less.

Find the first derivative with respect to x

$$f'(x) = (3x^3 - x^2 + 4x - 2)' = 3(3x^2) - 2x + 4(1) - 0 = 9x^2 - 2x + 4$$

Find the critical numbers

$$f'(x) = 0 \Longrightarrow 9x^2 - 2x + 4 = 0$$

Find the discriminant

$$D = (-2)^2 - 4(9)(4) = -140 < 0$$
$$coefficient = 9 > 0$$

Since the discriminant is less than zero and coefficient is greater than zero the expression $9x^2 - 2x + 4$ is positive for all real x.

The first derivative of the function exists in the set of real numbers and does not vanish. The function $f(x) = 3x^3 - x^2 + 4x - 2$ has no relative extreme points.