Answer on Question #76532 – Math – Calculus

1. Find the values of the following derivatives. Use

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Question

a) $\frac{ds}{dt}|_{t=-1} \quad if \quad s = 1 - 3t^2$

Solution

$$\frac{ds}{dt}|_{t=-1} = \lim_{h \to 0} \frac{s(-1+h) - s(-1)}{h} = \lim_{h \to 0} \frac{1 - 3(-1+h)^2 - (1 - 3(-1)^2)}{h} =$$

$$= \lim_{h \to 0} \frac{1 - 3 + 6h - 3h^2 - 1 + 3}{h} = \lim_{h \to 0} \frac{6h - 3h^2}{h} = \lim_{h \to 0}(6 - 3h) = 6 - 0 = 6.$$

Question

b) $\frac{dy}{dx}|_{x=\sqrt{3}} \quad if \quad y = -\frac{1}{x}$

Solution

$$\frac{dy}{dx}|_{x=\sqrt{3}} = \lim_{h \to 0} \frac{y\left(\sqrt{3} + h\right) - y(\sqrt{3})}{h} = \lim_{h \to 0} \frac{-\frac{1}{\sqrt{3}+h} - \left(-\frac{1}{\sqrt{3}}\right)}{h} =$$

$$= \lim_{h \to 0} \frac{\sqrt{3} + h - \sqrt{3}}{h\sqrt{3}(\sqrt{3}+h)} = \lim_{h \to 0} \frac{h}{h\sqrt{3}(\sqrt{3}+h)} = \lim_{h \to 0} \frac{1}{\sqrt{3}(\sqrt{3}+h)} = \frac{1}{\sqrt{3}(\sqrt{3}+0)} = \frac{1}{3}.$$

Question

c) $\frac{dr}{d\theta}|_{\theta=0} \quad if \quad r = \frac{2}{\sqrt{4-\theta}}$

Solution

$$\frac{dr}{d\theta}|_{\theta=0} = \lim_{h \to 0} \frac{r(0+h) - r(0)}{h} = \lim_{h \to 0} \frac{\frac{2}{\sqrt{4-(0+h)}} - \frac{2}{\sqrt{4-0}}}{h} =$$

$$= 2\lim_{h \to 0} \frac{\sqrt{4} - \sqrt{4-h}}{h\sqrt{4}\sqrt{4-h}} = \lim_{h \to 0} \left(\frac{\sqrt{4} - \sqrt{4-h}}{h\sqrt{4-h}} \cdot \frac{\sqrt{4} + \sqrt{4-h}}{\sqrt{4} + \sqrt{4-h}}\right) =$$

$$= \lim_{h\to 0} \frac{4-(4-h)}{h\sqrt{4-h}(\sqrt{4}+\sqrt{4-h})} = \lim_{h\to 0} \frac{h}{h\sqrt{4-h}(\sqrt{4}+\sqrt{4-h})} =$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{4-h}(\sqrt{4}+\sqrt{4-h})} = \frac{1}{\sqrt{4-0}(\sqrt{4}+\sqrt{4-0})} = \frac{1}{8}$$

Question

d) $\frac{dw}{dz}|_{z=4} \quad if \quad w = z+\sqrt{z}$

Solution

$$\frac{dw}{dz}|_{z=4} = \lim_{h\to 0} \frac{w(4+h)-w(4)}{h} = \lim_{h\to 0} \frac{4+h+\sqrt{4+h}-(4+\sqrt{4})}{h} =$$

$$= \lim_{h\to 0} \left(1+\frac{\sqrt{4+h}-\sqrt{4}}{h}\right) = 1+\lim_{h\to 0} \left(\frac{\sqrt{4+h}-\sqrt{4}}{h}\cdot\frac{\sqrt{4+h}+\sqrt{4}}{\sqrt{4+h}+\sqrt{4}}\right) =$$

$$= 1+\lim_{h\to 0} \frac{4+h-4}{h(\sqrt{4+h}+\sqrt{4})} = 1+\lim_{h\to 0} \frac{h}{h(\sqrt{4+h}+\sqrt{4})} =$$

$$= 1+\lim_{h\to 0} \frac{1}{\sqrt{4+h}+\sqrt{4}} = 1+\frac{1}{\sqrt{4+0}+\sqrt{4}} = \frac{5}{4}$$

Answer: a) 6; **b)** $\frac{1}{3}$; **c)** $\frac{1}{8}$; **d)** $\frac{5}{4}$.