Answer on Question #76449, Math / Calculus

Let $f(x, y) = \dfrac{x^2 y}{x^4 + y^2}$, if $x^4 + y^2 \neq 0$

Examine whether $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ exist or not.

Solution

Path $y = x$

$$\lim_{(x,x)\to(0,0)} f(x,x) = \lim_{(x,x)\to(0,0)} \frac{x^2(x)}{x^4 + x^2} = \lim_{(x,x)\to(0,0)} \frac{x}{x^2 + 1} = 0$$

Path $y = x^2$

$$\lim_{(x,x^2)\to(0,0)} f(x,x^2) = \lim_{(x,x)\to(0,0)} \frac{x^2(x^2)}{x^4 + x^4} = \lim_{(x,x)\to(0,0)} \frac{1}{2} = \frac{1}{2}$$

We find two curves $y = x$ and $y = x^2$ along which the limit does not agree.
Since the two limits are not the same, we can conclude that

$$\lim_{(x,y)\to(0,0)} f(x,y) \text{ does not exist}$$