*Question #76446, Math / Calculus*

Find the point on the ellipse,(x^2/4)+y^2=1
that is nearest to the origin.

*Solution*

We are dealing with the equation of the eclipse.

1) Rewrite x,y in polar coordinates: $x = r * Cos[\, a\, ],\ y = r * Sin[\, a\, ]$.

We are looking for minimal values of $r^2$( squared distance from the origin ).

2) The equation now take a form: $r^2 * \left(\frac{Cos[a]^2}{4} + Sin[a]^2\right) = 1$,

so $r^2 = \frac{1}{Cos[a]^2+4*Sin[a]^2} = \frac{4}{1+3*Sin[a]^2}$ and is defined and differentiable for any angles from $0\ to\ 2\pi$.

3) To find the minimum differentiate $r^2$. $\frac{d(r^2)}{da} = \frac{-24 * Sin[a]*Cos[a]}{(1+3*Sin[a]^2)^2}$.

Its derivative has zeroes at 0, $\frac{\pi}{2}$, $\pi$ and $\frac{3\pi}{2}$. The minimal value of $r^2$ should be at one or several points from this list.

4)

$$r^2(\,0\,) = 4,$$

$$r^2\left(\frac{\pi}{2}\right) = 1,$$

$$r^2(\pi) = 4,$$

$$r^2\left(\frac{3\pi}{2}\right) = 1.$$

So we see that the same minimal value take place at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, which corresponds to (0,1) and $(\,0,-1\,)$.

Answer: $(\,0,1\,)\ OR\ (\,0,-1\,)$.