Answer on Question #76445 – Math – Calculus

Question

Find the mass of an object which is in the form of a cuboid [0,1]×[2,4]×[1,3] The density at any point (x,y,z) on the cuboid is given by del (x,y,z)= x^2+y^2+z^2.

Solution

By definition, the mass of an object is given by integral

$$M = \int_V \rho(x, y, z) dxdydz.$$

In our case:

$\rho(x, y, z) = x^2 + y^2 + z^2; \quad 0 \le x \le 1; \; 2 \le y \le 4; \; 1 \le z \le 3.$

$$M = \int_0^1 dx \int_2^4 dy \int_1^3 dz \, (x^2 + y^2 + z^2) = \int_0^1 dx \int_2^4 dy \left[x^2 z + y^2 z + \frac{z^3}{3}\right]_1^3$$

$$= \int_0^1 dx \int_2^4 dy \left[(3-1)(x^2+y^2) + \left(\frac{27}{3} - \frac{1}{3}\right)\right] = \int_0^1 dx \int_2^4 dy \left[2(x^2+y^2) + \frac{26}{3}\right]$$

$$= \int_0^1 dx \left[2\left(x^2 y + \frac{y^3}{3}\right) + \frac{26}{3} y\right]_2^4 = \int_0^1 dx \left[2\left((4-2)x^2 + \left(\frac{64}{3} - \frac{8}{3}\right)\right) + \frac{26}{3}(4-2)\right]$$

$$= \int_0^1 dx \left[2\left(2x^2 + \frac{56}{3}\right) + \frac{52}{3}\right] = \int_0^1 dx \left[2\left(2x^2 + \frac{56}{3}\right) + \frac{52}{3}\right] = \int_0^1 dx \left[4x^2 + \frac{164}{3}\right]$$

$$= \left[\frac{4}{3}x^3 + \frac{164}{3}x\right]_0^1 = \frac{4}{3}(1-0) + \frac{164}{3}(1-0) = \frac{168}{3} = 56.$$

So the mass is $M = 56$.

Answer: 56.