Answer on Question#76443 – Math – Calculus

If $z = e^{(xy^2)}$, $x = t\cos(t)$, $y = t\sin(t)$ then find $\frac{dz}{dt}$, when $t = \frac{\pi}{2}$.

Solution

First of all, we will substitute expressions for x and y into z:

$$z = e^{\left(t\cos(t)(t\sin(t))^2\right)} = e^{(t^3\cos(t)\sin(t)^2)}$$

Now, we can find derivative $\frac{dz}{dt}$:

$$\frac{dz}{dt} = e^{(t^3\cos(t)\sin(t)^2)}\left(3t^2\cos(t)\sin(t)^2 - t^3\sin(t)\sin(t)^2 + 2t^3\cos(t)\sin(t)\cos(t)\right)$$
$$= t^2\sin(t)e^{(t^3\cos(t)\sin(t)^2)}(3\cos(t)\sin(t) - t\sin(t)^2 + 2t\cos(t)^2)$$
$$= t^2\sin(t)e^{(t^3\cos(t)\sin(t)^2)}\left(3\cos(t)\sin(t) + t(\cos(2t) + \cos(t)^2)\right).$$

And then substitute $t = \frac{\pi}{2}$:

$$\frac{dz}{dt} = \left(\frac{\pi}{2}\right)^2 \sin\left(\frac{\pi}{2}\right) e^{\left(\left(\frac{\pi}{2}\right)^3\cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)^2\right)}\left(3\cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right) + \frac{\pi}{2}\left(\cos\left(2\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right)^2\right)\right)$$
$$= \frac{\pi^2}{4}\left(\frac{\pi}{2}(-1)\right) = -\frac{\pi^3}{8}.$$

Answer:

$\frac{dz}{dt} = -\frac{\pi^3}{8}$.

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