Answer on Question #76442 – Math – Calculus

Question

If

$$f(x,y)=\begin{cases}x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right), & x\neq 0, y\neq 0\\ 0, & x=0, y=0\end{cases}$$

then show that

$$\frac{\partial^2 f}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}$$

Solution

If we want to calculate the partial derivative of $f(x,y)$ at any point away from the origin $(0,0)$ we can use

$$\frac{\partial f}{\partial y}|_{(x,y)\neq(0,0)}=x^2\left(\frac{1}{1+\frac{y^2}{x^2}}\right)\left(\frac{1}{x}\right)-2y\tan^{-1}\left(\frac{x}{y}\right)-y^2\left(\frac{1}{1+\frac{x^2}{y^2}}\right)\left(-\frac{x}{y^2}\right)=$$

$$=\frac{x^3+xy^2}{x^2+y^2}-2y\tan^{-1}\left(\frac{x}{y}\right)=x-2y\tan^{-1}\left(\frac{x}{y}\right)$$

$$\frac{\partial^2 f}{\partial x\partial y}|_{(x,y)\neq(0,0)}=1-2y\left(\frac{1}{1+\frac{y^2}{x^2}}\right)\left(\frac{1}{y}\right)=1-\frac{2y^2}{x^2+y^2}=\frac{x^2-y^2}{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial x\partial y}|_{(x,y)=(0,0)}=0$$