Answer on Question #76440 – Math – Calculus

Question

If possible then find the value of f such that

$$\boldsymbol{F} = (4x^3 + 9x^2y^2, 6x^3y + 6y^5) = \nabla f$$

Solution

$\boldsymbol{F}(x, y) = (4x^3 + 9x^2y^2)\boldsymbol{i} + (6x^3y + 6y^5)\boldsymbol{j}$.

Here $P = 4x^3 + 9x^2y^2$ and $Q = 6x^3y + 6y^5$, so

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(4x^3 + 9x^2y^2) = 18x^2y \quad \text{and} \quad \frac{\partial Q}{\partial x} = \frac{\partial}{\partial y}(6x^3y + 6y^5) = 18x^2y$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

Since the domain of $\boldsymbol{F}$ is all of $\boldsymbol{R}^2$, we conclude that $\boldsymbol{F}$ is conservative. Thus there exists a function f such that $\nabla f = \boldsymbol{F}$.

$$f_x(x, y) = 4x^3 + 9x^2y^2, \quad f_y(x, y) = 6x^3y + 6y^5$$

Integrate first equation with respect to x

$$f(x, y) = 4\left(\frac{x^4}{4}\right) + 9y^2\left(\frac{x^3}{3}\right) + g(y)$$

$$f(x, y) = x^4 + 3x^3y^2 + g(y)$$

Thus

$$f_y(x, y) = 3x^3(2y) + g'(y)$$

But

$$f_y(x, y) = 6x^3y + 6y^5$$

So

$$3x^3(2y) + g'(y) = 6x^3y + 6y^5$$

$$g'(y) = 6y^5$$

$$g(y) = 6\left(\frac{y^6}{6}\right) + C$$

$$g(y) = y^6 + C$$

Therefore,

$$f(x, y) = x^4 + 3x^3y^2 + y^6 + C$$

Answer: $f(x, y) = x^4 + 3x^3y^2 + y^6 + C$.