Answer on Question #76438 – Math – Calculus

Question

Find the value of a, b, c such that $\lim\limits_{x\to\infty} \frac{ae^x+bcosx+ce^{-x}}{xsinx} = \frac{3}{2}$.

Conjecture

$$\lim_{x\to\infty} \frac{ae^x+bcosx+ce^{-x}}{xsinx} \neq \frac{3}{2}\ for\ any\ a, b, c \in R.$$

Proof (by contradiction)

Let $f(x) = \frac{ae^x+bcosx+ce^{-x}}{xsinx}$.

Suppose that above-mentioned limit exists and $\lim\limits_{x\to\infty} f(x) = \frac{3}{2}$ then according to the definition of a limit of the function by Eduard Heine

$$\forall \{x_n\}_{n=1}^{\infty}: \lim_{n\to\infty} x_n = \infty => \lim_{n\to\infty} f(x_n) = \frac{3}{2}.$$

Let's consider $x_n = \frac{\pi}{2} + 2\pi n;$

$$f(x_n) = \frac{ae^{(\frac{\pi}{2}+2\pi n)}+ce^{-(\frac{\pi}{2}+2\pi n)}}{\frac{\pi}{2}+2\pi n};$$

$$\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} \frac{ae^{(\frac{\pi}{2}+2\pi n)}+ce^{-(\frac{\pi}{2}+2\pi n)}}{\frac{\pi}{2}+2\pi n} = \begin{cases} a\cdot(+\infty),\ a \neq 0 \\ 0,\ a = 0 \end{cases}$$

because of the fact that

$$\lim_{x\to+\infty} \frac{e^x}{x} = [according\ to\ the\ Hospital's\ rule] = \lim_{x\to+\infty} \frac{(e^x)'}{x'} = \lim_{x\to+\infty} e^x = +\infty.$$

Here is the contradiction ■

Answer: $\lim\limits_{x\to\infty} \frac{ae^x+bcosx+ce^{-x}}{xsinx} \neq \frac{3}{2}\ for\ any\ a, b, c \in R.$