Answer on Question #76433 – Math – Statistics and Probability

Question

A random variable X has following probability distribution:

$$f(x) = \begin{cases} cx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of c and $Var(X)$.

Solution

From the probability density function (PDF) we can determine the value of c by integrating the PDF and setting it equal to 1

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^3 cx^2\,dx = \left[c\left(\frac{x^3}{3}\right)\right]\begin{matrix}3\\0\end{matrix} = \frac{c(3)^3}{3} - 0 = 9c = 1$$

$$c = \frac{1}{9}$$

According to the formulae

$$Var(X) = E(X^2) - (E(X))^2,$$

where

$$E(X) = \int_{-\infty}^{\infty} xf(x)\,dx = \int_0^3 cx^3\,dx = \int_0^3 \frac{1}{9}x^3\,dx = \frac{1}{9}\left[\frac{x^4}{4}\right]\begin{matrix}3\\0\end{matrix} = \frac{(3)^4}{9(4)} - 0 = \frac{9}{4}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^3 cx^4\,dx = \int_0^3 \frac{1}{9}x^4\,dx = \frac{1}{9}\left[\frac{x^5}{5}\right]\begin{matrix}3\\0\end{matrix} = \frac{(3)^5}{9(5)} - 0 = \frac{27}{5}$$

$$Var(X) = E(X^2) - \big(E(X)\big)^2 = \frac{27}{5} - \left(\frac{9}{4}\right)^2 = \frac{432 - 405}{80} = \frac{27}{80}$$

Answer: $c = \frac{1}{9}$, $Var(X) = \frac{27}{80}$.