Answer on Question #76430 – Math – Calculus

Question

Obtain the curl of the following vector field:

$$A = e_r + r\cos\theta\, e_\theta + re_\varphi$$

Solution

Obtain the curl of the vector field

$$\operatorname{curl}\vec{A} = \vec{\nabla}\times\vec{A} = \left(e_r\frac{\partial}{\partial r} + \frac{e_\theta}{r}\frac{\partial}{\partial\theta} + \frac{e_\varphi}{r\sin\theta}\frac{\partial}{\partial\varphi}\right)\times\left(A_r e_r + A_\theta e_\theta + A_\varphi e_\varphi\right) =$$

$$= \frac{e_r}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(A_\varphi\sin\theta\right) - \frac{\partial A_\theta}{\partial\varphi}\right] + \frac{e_\theta}{r\sin\theta}\left[\frac{\partial A_r}{\partial\varphi} - \sin\theta\frac{\partial}{\partial r}\left(rA_\varphi\right)\right] +$$

$$+\frac{e_\varphi}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right] = \frac{e_r}{r\sin\theta}[r\cos\theta - 0] + \frac{e_\theta}{r\sin\theta}[0 - \sin\theta\,(2r)] +$$

$$+\frac{e_\varphi}{r}[2r\cos\theta - 0] = \cot\theta\, e_r - 2e_\theta + 2\cos\theta\, e_\varphi.$$

Answer: $\operatorname{curl}\vec{A} = \vec{\nabla}\times\vec{A} = \cot\theta\, e_r - 2e_\theta + 2\cos\theta\, e_\varphi$.