Answer on Question #76428 – Math – Calculus

Question

Show that for two scale field f and g : ∇×(f ∇g)+∇×(g∇f)=0.

Solution

The gradient of the product of two scalar fields:

$$\nabla(f \cdot g) = g\nabla f + f\nabla g$$

then:

$$\nabla \times (f\nabla g) = \nabla \times (\nabla(f \cdot g) - g\nabla f) = \nabla \times \big(\nabla(f \cdot g)\big) - \nabla \times (g\nabla f)$$

The curl of the gradient of any scalar field is always the zero vector field:

$$\nabla \times (\nabla(f \cdot g)) = 0$$

then

$$\nabla \times (f\nabla g) + \nabla \times (g\nabla f) = \nabla \times \big(\nabla(f \cdot g)\big) - \nabla \times (g\nabla f) + \nabla \times (g\nabla f) = \nabla \times \big(\nabla(f \cdot g)\big) = 0$$