Answer on Question #76421 – Math – Calculus

Question

Find the area enclosed by curve r=a(1-cos theta).

Solution

The enclosed area by curve $r = a(1 - \cos\theta)$ on a polar coordinate system can be evaluated by the following formula:

$$S = \frac{1}{2}\int_0^{2\pi} r^2 d\theta = \frac{1}{2}\int_0^{2\pi} (a(1 - \cos\theta))^2 d\theta.$$

Since the curve $r = a(1 - \cos\theta)$ is a symmetric relative to the polar axis, then we have

$$S = \frac{1}{2}\cdot 2\int_0^{\pi} (a(1 - \cos\theta))^2 d\theta = \int_0^{\pi} a^2(1 - \cos\theta)^2\, d\theta = a^2\int_0^{\pi} (1 - 2\cdot\cos\theta + \cos^2\theta)\, d\theta =$$

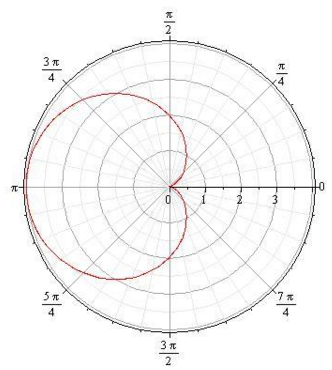
$$= a^2\left[\int_0^{\pi} d\theta - \int_0^{\pi} 2\cdot\cos\theta\, d\theta + \int_0^{\pi} \cos^2\theta\, d\theta\right] =$$

$$= a^2\left[\theta|_0^{\pi} - 2\cdot\sin\theta\,|_0^{\pi} + \int_0^{\pi} \frac{1 + \cos 2\theta}{2} d\theta\right] =$$

$$= a^2\left[(\pi - 0) - 2\cdot(\sin\pi - \sin 0) + \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)|_0^{\pi}\right] =$$

$$= a^2\left[\pi + \left(\frac{\pi}{2} + \frac{\sin 2\pi}{4}\right) - \left(\frac{0}{2} + \frac{\sin 0}{4}\right)\right] == a^2\left[\pi + \frac{\pi}{2}\right] = \frac{3\pi a^2}{2}.$$

Below we give an intuitive plot of curve $r = a(1 - \cos\theta)$ for the case of constant a=2.



Answer: $\frac{3\pi a^2}{2}$.

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