Answer on Question #76037 – Math – Differential Equations

Question

Solve the differential equation.
Suppose the temperature of a body when discovered is 85° F. Two hours later, the temperature is 74°F and the room temperature is 68°F. Find the time when the body was discovered after death (assume the body temperature to be 98.6°F at the time of death.)

Solution

Using Newton's Law of Cooling tells us:

$\frac{dT}{dt} = k(68 - T)$, T_0=98.6, $T(t_1)=T_1$=85, T_2=T(t1+2)=74.

$$\frac{dT}{dt} = k(68 - T), \quad \frac{dT}{dt} = -k(T - 68), \qquad \frac{dT}{T - 68} = -kdt.$$

Now we can integrate both sides of equations, variables are separated.

$$\int \frac{dT}{T - 68} = \int -kdt$$

$$\ln(T - 68) = -kt + C.$$

$$T - 68 = C\,e^{-kt}, \qquad T = 68 + Ce^{-kt}.$$

We can use initial condition to find the value of C:

$$T(0) = 68 + C = 98.6, \quad C = 98.6 - 68 = 30.6.$$

The solution of this differential equation is $T = 68 + 30.6e^{-kt}$.

Now we need to find the value of k.

$$85 = 68 + 30.6e^{-kt_1}, \qquad 30.6e^{-kt_1} = 17, \qquad e^{-kt_1} = \frac{17}{30.6}$$

$$74 = 68 + 30.6e^{-k(t_1+2)}, \qquad 30.6e^{-kt_1-2k} = 6, \qquad 30.6e^{-kt_1}e^{-2k} = 6.17e^{-2k} = 6, \qquad -2k = \ln\frac{6}{17},$$

$$k = \frac{1}{2}\ln\frac{17}{6}.$$

Now we can find t_1:

$$e^{-kt_1} = \frac{17}{30.6}, \qquad -kt_1 = \ln\frac{17}{30.6}, \qquad t_1 = \frac{1}{k}\ln\frac{30.6}{17} = 2\frac{\ln\frac{30.6}{17}}{\ln\frac{17}{6}} = 1.13\ (hours).$$

Answer: body was found in 1.13 hours after death.