Answer on Question #75837 – Math – Calculus

Question

Obtain the Fourier series expansion for the following periodic function which has a period of π: f(x)={(4/π)x for 0<x<π/2 (-4/π)x for (-π/2)<x<0

Solution

$$f(x) = \begin{cases} \frac{4}{\pi}x, & 0 \le x < \pi/2, \\ -\frac{4}{\pi}x, & -\frac{\pi}{2} \le x < 0, \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n cosnx + b_n sinnx)$$

$$a_0 = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{4}{\pi}xdx - \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{0}\frac{4}{\pi}xdx = \frac{2}{\pi^2}x^2\Big|_{x=0}^{x=\frac{\pi}{2}} - \frac{2}{\pi^2}x^2\Big|_{x=-\frac{\pi}{2}}^{x=0} = 1.$$

$$a_n = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)cosnx\,dx = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{4}{\pi}xcosnx\,dx - \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{0}\frac{4}{\pi}xcosnx\,dx =$$

$$= \frac{4}{\pi^2}\left(\frac{x}{n}\ sinnx + \frac{1}{n^2}cosnx\right)\Big|_{x=0}^{x=\frac{\pi}{2}} - \frac{4}{\pi^2}\left(\frac{x}{n}\ sinnx + \frac{1}{n^2}cosnx\right)\Big|_{x=-\frac{\pi}{2}}^{x=0} =$$

$$= \frac{2}{\pi^2}\left(\frac{\pi}{2n}sin\frac{\pi n}{2} + \frac{1}{n^2}cos\frac{\pi n}{2} - \frac{1}{n^2}\right).$$

$$b_n = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)sinnx\,dx = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{4}{\pi}xsinnx\,dx - \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{0}\frac{4}{\pi}xsinnx\,dx =$$

$$= \frac{4}{\pi^2}\left(-\frac{x}{n}\ cosnx + \frac{1}{n^2}sinnx\right)\Big|_{x=0}^{x=\frac{\pi}{2}} - \frac{4}{\pi^2}\left(-\frac{x}{n}\ cosnx + \frac{1}{n^2}sinnx\right)\Big|_{x=-\frac{\pi}{2}}^{x=0} = 0$$

$$f(x) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{\pi}{2n}sin\frac{\pi n}{2} + \frac{1}{n^2}cos\frac{\pi n}{2} - \frac{1}{n^2}\right)cosnx\,.$$