Answer on Question #75735 - Subject – Differential Equation

Given: $f(x,y) = x^2 \sin y + y^2 \cos x$

To find: All the first and second order partial differential equations

Solution: Consider $f(x,y) = x^2 \sin y + y^2 \cos x$

Differentiating partially with respect to x and y, we get

$$\frac{\partial f}{\partial x} = 2x \sin y - y^2 \sin x \quad \text{and} \quad \frac{\partial f}{\partial y} = x^2 \cos y + 2y \cos x$$

$$\therefore \quad x\frac{\partial f}{\partial x} = 2x^2 \sin y - xy^2 \sin x \quad \text{and} \quad y\frac{\partial f}{\partial y} = x^2 y \cos y + 2y^2 \cos x$$

$$\Rightarrow \quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2x^2 \sin y - xy^2 \sin x + x^2 y \cos y + 2y^2 \cos x$$

$$\Rightarrow \quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2(x^2 \sin y + y^2 \cos x) - xy(y \sin x - x \cos y)$$

$$\Rightarrow \quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f - xy(y \sin x - x \cos y) \qquad \text{...... (i)}$$

Again differentiating,

$$\frac{\partial^2 f}{\partial x^2} = 2\sin y - y^2 \cos x, \qquad \frac{\partial^2 f}{\partial y^2} = -x^2 \sin y + 2\cos x \quad \text{and} \quad \frac{\partial^2 f}{\partial x \partial y} = 2(x \cos y - y \sin x)$$

From equation (i)

$$y \sin x - x \cos y = \frac{1}{xy}\left(2f - x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y}\right)$$

$$\Rightarrow \quad x \cos y - y \sin x = \frac{1}{xy}\left(x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} - 2f\right)$$

$$\Rightarrow \quad \frac{\partial^2 f}{\partial x^2} = \frac{1}{xy}\left(x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} - 2f\right) \qquad \text{...... (ii)}$$

using second order derivative.