ANSWER on Question #75734 – Math – Differential Equations

Show that function

$$u(x,t) = e^{-\mu t}\sin x$$

is a solution of one dimensional heat equation.

SOLUTION

By the definition, the one dimensional heat equation has form

$$\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial x^2}$$

(More information: https://en.wikipedia.org/wiki/Heat_equation)

Then, we calculate the above partial derivatives of the proposed solution

$$u(x,t) = e^{-\mu t}\sin x \to \frac{\partial u}{\partial t} = \frac{\partial(e^{-\mu t}\sin x)}{\partial t} = \frac{d(e^{-\mu t})}{dt}\sin x = -\mu e^{-\mu t}\sin x$$

$$\boxed{\frac{\partial u}{\partial t} = -\mu e^{-\mu t}\sin x}$$

$$u(x,t) = e^{-\mu t}\sin x \to \frac{\partial u}{\partial x} = \frac{\partial(e^{-\mu t}\sin x)}{\partial x} = e^{-\mu t}\frac{d(\sin x)}{dx} = e^{-\mu t}\cos x \to$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}(e^{-\mu t}\cos x) = e^{-\mu t}\frac{d(\cos x)}{dx} = -e^{-\mu t}\sin x$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -e^{-\mu t}\sin x}$$

Conclusion,

$$u(x,t) = e^{-\mu t}\sin x \to \begin{cases} \dfrac{\partial u}{\partial t} = -\mu e^{-\mu t}\sin x \\ \dfrac{\partial^2 u}{\partial x^2} = -e^{-\mu t}\sin x \end{cases}$$

Then,

$$\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2} \leftrightarrow -\mu e^{-\mu t} \sin x = \mu \cdot (-e^{-\mu t} \sin x)$$

This means that this function $u(x,t)$ is indeed a solution of the one-dimensional heat equation.