Answer on Question #75566 – Math – Calculus

Question

Show that the circular cylinder S={(x,y,z)∈R^3 |x^2+y^2=1} can be covered by a single surface patch and so a surface.

Solution

We can take U an annulus instead of a disc where $U = \{(u, v): 0 < u^2 + v^2 < \pi\}$. Any point in the annulus U is uniquely of the form $(t\cos\theta, t\sin\theta)$ for some real $t \in (0, \sqrt{\pi}), \theta \in [0, 2\pi)$. Map this point to the point of the cylinder $(x, y, z) = (\cos\theta, \sin\theta, \cot t^2)$. This is clearly a subset of the cylinder as it satisfies $x^2 + y^2 = 1$. Also, because θ ranges in $[0, 2\pi)$, for any fixed z the entire slice of the cylinder at that z level gets covered. Because the cotangent of t^2 for $t \in (0, \sqrt{\pi})$ takes on every real value, every level z indeed gets hit, showing the result of mapping the annulus as above covers the whole cylinder.

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