Answer on Question #75565 – Math – Calculus

Question

The hyperboloid of one sheet is

$$S = \{(x, y, z) \in R^3 \,|x^2 + y^2 - z^2 = 1\}$$

Show that for every θ, the straight line

$$(x - z)cos\theta = (1 - y)sin\theta, (x + z)sin\theta = (1 + y)cos\theta$$

is contained in S and that every point of hyperboloid lies on one of these lines. Deduce that S can be covered by a single surface patch, and hence is a surface.

Solution

Multiplying two equations of lines we get:

$$(x - z)cos\theta(x + z)sin\theta = (1 - y)sin\theta(1 + y)cos\theta$$

$$(x - z)(x + z) = (1 - y)(1 + y)$$

$$x^2 - z^2 = 1 - y^2$$

$$x^2 + y^2 - z^2 = 1$$

So, the line is contained in S and every point of hyperboloid lies on one of the given lines.

The Jacobian of $f(x, y, z) = x^2 + y^2 - z^2 - 1$ is $(0,0)$ only in $(0,0)$ so it is a smooth surface. You can move the lines continuously to go from a point $p \in S$ to any point q so it is a single surface patch.