

Question #75452, Math / Algebra

Find the roots of the equation $2x^3 - x^2 - 22x - 24 = 0$ if two of them are in the ratio 3:4.

Answer.

Let the roots of the given cubic equation are $3a, 4a$ and b .

Thus we have

$$3a + 4a + b = -\frac{-24}{2} = \frac{1}{2}$$

$$12a^2 + 4ab + 3ab = -\frac{22}{2} = -11$$

$$3a(4a)b = -\frac{-24}{2} = 12$$

Then

$$\begin{cases} 7a + b = \frac{1}{2} \\ 12a^2 + 7ab = -11 \\ a^2b = 1 \end{cases}$$

Eliminating b we get: $b = \frac{1}{2} - 7a$,

$$12a^2 + 7a\left(\frac{1}{2} - 7a\right) = -11 \text{ or}$$

$$74a^2 - 7a - 22 = 0 \rightarrow a = \frac{7 \pm \sqrt{49 + 6512}}{148} = \frac{7 \pm 81}{148} \rightarrow$$

$$\rightarrow a = -\frac{1}{2}, \quad a = \frac{22}{37},$$

$$b = \frac{1}{2} - 7a \rightarrow b = 4, \quad b = -\frac{171}{74}.$$

Since $\frac{3}{11}, \frac{4}{11}$ and $-\frac{171}{74}$ are not the roots, the roots are $-\frac{3}{2}, -2, 4$.

