Answer on Question #75432 – Math – Calculus
Question

$f(x,y) = 5xy - ln(xy) - 5$, check whether there exists a continuously differentiable function g defined by $f(x,y) = 0$ in the neighbourhood of $x = 3$, such that $g(3) = 1/3$. Find $g'(3)$ if exists.

Solution

We have

$$f(x,y) = 5xy - ln(xy) - 5 = 0$$

Then

$$f\left(3,\frac{1}{3}\right) = 5\cdot 3\cdot\frac{1}{3} - \ln\left(3\cdot\frac{1}{3}\right) - 5 = 5 - \ln 1 - 5 = 0$$

If we put $y = g(x)$, then we have $g(3) = 1/3$.

Also

$$f(x,y) = 5xg(x) - \ln\big(xg(x)\big) - 5 = 0$$

$$5g(x) + 5xg'(x) - \frac{g(x) + xg'(x)}{xg(x)} = 0$$

$$g'(x)\left(5x - \frac{1}{g(x)}\right) = \frac{1}{x} - 5g(x)$$

$$g'(x) = \frac{\frac{1}{x} - 5g(x)}{5x - \frac{1}{g(x)}}$$

Then

$$g'(3) = \frac{\frac{1}{3} - 5g(3)}{5\cdot 3 - \frac{1}{g(3)}} = \frac{\frac{1}{3} - 5\cdot\frac{1}{3}}{5\cdot 3 - 3} = -\frac{4}{3\cdot 12} = -\frac{1}{9}.$$

We can conclude that there exists a continuously differentiable function $g(x)$ defined by $f(x,y) = 0$ in the neighbourhood of $x = 3$, such that $g(3) = 1/3$.

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