Find $\frac{dy}{dx}$ for the function $x = a\cos^3 t, y = a\sin^3 t$.

Solution

Use the formula of the derivative of parametric functions:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad (1)$$

where

$$\frac{dy}{dt} = 3a\sin^2 t\cos t, (2)$$

$$\frac{dx}{dt} = -3a\cos^2 t\sin t. (3)$$

Let's substitute (2), (3) in (1):

$$\frac{dy}{dx} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = -\frac{\sin t}{\cos t} = -\tan t$$

Answer: $\frac{dy}{dx} = -\tan t$.