Question

Using sin(0.1)= 0.09983 and sin(0.2)= 0.19867, find an approximate value of sin(0.15) by using Lagrange's interpolation. Obtain a bound on the truncation error.

Solution

We have

$$P_1(0.15) = \frac{0.2 - 0.15}{0.2 - 0.1}(0.09983) + \frac{0.15 - 0.1}{0.2 - 0.1}(0.19867) = (0.5)(0.09983) + (0.5)(0.19867) = 0.14925$$

The truncation error is

$$E_1(f; x) = \frac{(x - 0.1)(x - 0.2)}{2}(-\sin \xi),$$

$$0.1 < \xi < 0.2$$

The maximum value of $|-\sin \xi|$, $0.1 < \xi < 0.2$ is $\sin 0.2 = 0.19867$. Thus,

$$|E_1(f; x)| \leq \left| \frac{(0.15 - 0.1)(0.15 - 0.2)}{2} \right| (0.19867) = (0.00125)(0.19867) \approx 0.00025.$$