Question #75299, Math / Other

find the minimum number of intervals required to evaluate integration 0 to 1 e^-x^2 dx with an accuracy
of 1/2 X 10^-4 by using trapezoidal rule

Answer.

$$|\varepsilon| \leq \frac{(b-a)^3}{12N^2} \max_{a \leq x \leq b} f''(x).$$

$$a = 0, \ b = 1.$$

$$f(x) = e^{-x^2}, \ f'(x) = -2xe^{-x^2}, \ f''(x) = (4x^2 - 2)e^{-x^2}.$$

$$\max_{0 \leq x \leq 1} f''(x) = f''(1) = \frac{2}{e} \approx 0.7358.$$

So, $0.5 \times 10^{-4} \leq \frac{0.7358}{12N^2} \quad \rightarrow \quad N \geq \sqrt{\frac{0.7358}{12*0.5 \times 10^{-4}}} \approx 35.$