Answer on Question #75288- Math – Differential Equations

**Identifying the differential equation and solve them $y = xy' + 1 - lny'$.**

## Solution

This equation of a look:

$$y(x) = x\frac{dy}{dx} + f\left(\frac{dy}{dx}\right)$$

is called **Clairaut's equation**.

Use the next replacement:

$$\frac{dy}{dx} = p,$$

then

$$y = xp + 1 - lnp \quad (1)$$

Let's differentiate

$$\frac{dy}{dx} = p + x\frac{dp}{dx} - \frac{1}{p}\cdot\frac{dp}{dx},$$

then

$$p = p + x\frac{dp}{dx} - \frac{1}{p}\cdot\frac{dp}{dx}$$

$$x\frac{dp}{dx} - \frac{1}{p}\cdot\frac{dp}{dx} = 0$$

$$\frac{dp}{dx}\left(x - \frac{1}{p}\right) = 0$$

Equate each multiplier to zero:

$$\frac{dp}{dx} = 0 \ (2)\ ;$$

$$x - \frac{1}{p} = 0 \quad (3).$$

Integrate equality (2):

$$p = C,$$

let's substitute this value in (1): $y = Cx + 1 - lnC$ - common decision.

From (3):

$$x - \frac{1}{p} = 0, x = \frac{1}{p}, p = \frac{1}{x}$$

let's substitute this value in (1):

$$y = x\frac{1}{x} + 1 - ln\frac{1}{x}$$

$$y = 1 + 1 - lnx^{-1}$$

$y = 2 + lnx$ - singular solution

**Answer:** $y = Cx + 1 - lnC, y = 2 + lnx.$