

Answer on Question #75245-Math-Other

Determine the direction in which the scalar field $\phi(x, y) = xy^2 + x^3y$ increases the fastest at the point (1, 2).

Solution

$$\nabla\phi = \frac{\partial\phi}{\partial x}\mathbf{i} + \frac{\partial\phi}{\partial y}\mathbf{j}$$

$$\nabla\phi(x, y) = (y^2 + 3x^2y)\mathbf{i} + (2xy + x^3)\mathbf{j}$$

$$\nabla\phi(1, 2) = (2^2 + 3(1^2)2)\mathbf{i} + (2(1)2 + 1^3)\mathbf{j} = 10\mathbf{i} + 5\mathbf{j}$$

$$|\nabla\phi| = \sqrt{10^2 + 5^2} = 5\sqrt{5}$$

The direction in which the scalar field $\phi(x, y)$ increases the fastest at the point (1, 2) is

$$\frac{\nabla\phi}{|\nabla\phi|} = \frac{10\mathbf{i} + 5\mathbf{j}}{5\sqrt{5}} = \frac{2\mathbf{i} + \mathbf{j}}{\sqrt{5}} = \frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j}$$

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