Given tan(A) = (3/4), 0 < A < (π/2) and cos(B) = (5/13), (3π/2) < B < 2π) determine cos(2A)

Solution

Let find $\cos A$. We will use formula:

$$1 + tan^2 \propto = \frac{1}{cos^2 \propto}$$

$$cos^2 \propto = \frac{1}{1 + tan^2 \propto}$$

$$cos^2 A = \frac{1}{1 + (\frac{3}{4})^2} = \frac{1}{1 + \frac{9}{16}} = \frac{1}{\frac{16 + 9}{16}} = \frac{16}{25}$$

$$cosA = \pm\frac{4}{5}$$

If 0<A<$\frac{\pi}{2}$ then $cosA = \frac{4}{5}$

Let find $\sin A$

We will use the formula:

$$sin^2 \propto + cos^2 \propto = 1$$

$$sin^2 A = 1 - cos^2 A = 1 - (\frac{4}{5})^2 = \frac{25 - 16}{25} = \frac{9}{25}$$

$$sinA = \pm\frac{3}{5}$$

If 0<A<$\frac{\pi}{2}$ then $sinA = \frac{3}{5}$

Let find $\cos 2A$

We will use the formula:

$$cos2 \propto = cos^2 \propto - sin^2 \propto$$

$$cos2A = (\frac{4}{5})^2 - (\frac{3}{5})^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Answer cos2A=$\frac{7}{25}$