Question #75223, Math / Other

find the minimum number of intervals required to evaluate integration 0 to 1 e^-x^2 dx with an accuracy of 1/2 X 10^-4 by using trapezoidal rule

Answer.

$$|\varepsilon| \le \frac{(b-a)^3}{12N^2} \max_{a \le x \le b} f''(x).$$

$$a = 0,\ b = 1.$$

$$f(x) = e^{-x^2},\ f'(x) = -2xe^{-x^2},\ f''(x) = (4x^2 - 2)e^{-x^2}.$$

$$\max_{0 \le x \le 1} f''(x) = f''(1) = \frac{2}{e} \approx 0.7358.$$

$$\text{So, } 0.5 \times 10^{-4} \le \frac{0.7358}{12N^2} \quad \rightarrow \quad N \ge \sqrt{\frac{0.7358}{12 * 0.5 \times 10^{-4}}} \approx 35.$$

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