Answer on Question #75213 – Math – Quantitative Methods

Question

The solution of the system of equations (1 2, 2 1)(x,y) =(4,-2) is attempt by the Gauss Jacobi and Gauss Seidel iteration schemes. Set up the two schemes in matrix form. Will the iteration schemes converge? Justify your answer.

Solution

Let the system of equations is given by

$$A\vec{x} = \vec{b}, \ where \quad A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \ and \quad \vec{b} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \ \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad (1)$$

The Gauss Jacobi method. The solution is then obtained iteratively through:

$$x^{k+1} = D^{-1}(b - Rx^k) \quad where \ A = D + R, \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

The convergence condition is that the spectral radius of the iteration matrix is less than 1:

$$\rho(D^{-1}R) < 1$$

$$D^{-1}R = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \rightarrow \rho(D^{-1}R) = 2$$

the Gauss Jacobi method does not converge for system (1)

The Gauss Seidel method. The solution is then obtained iteratively through:

$$x^{k+1} = (L + D)^{-1}(b - Ux^k) \quad where \ A = L + D + U, \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, L = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, U = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

The convergence condition:

$$\|-(L + D)^{-1}U\| < 1$$

$$-(L + D)^{-1}U = \begin{bmatrix} 0 & -2 \\ 0 & 4 \end{bmatrix} \rightarrow \|-(L + D)^{-1}U\| = 4$$

the Gauss Seidel method does not converge for system (1).

Answer: The solution of the system of equations (1 2, 2 1) (x, y) = (4, -2) by means of the iterative methods of Gauss Jacobi and Gauss Seidel does not converge.