Question #75212 - Math – Quantitative Methods

Starting with x^(0)=[1 1 1]^T, find the dominant eigenvalue and corresponding eigenvector for the matrix A= [4 -1 1, 4 -8 1, -2 1 5] using the power mwthod

Solution: the power method is described by the recurrence relation:

$$x_{k+1} = \frac{Ax_k}{\|Ax_k\|} = \frac{A^{k+1}x_0}{\|A^{k+1}x_0\|}$$

if $A = \begin{bmatrix} 4 & -1 & 1 \\ 4 & -8 & 1 \\ -2 & 1 & 5 \end{bmatrix}$ $and\ x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $then\ \ Ax_0 = \begin{bmatrix} 4 & -1 & 1 \\ 4 & -8 & 1 \\ -2 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 4 \end{bmatrix}$ $and\ \|Ax_0\| = 4$:

$$x_1 = \frac{1}{4} \cdot \begin{bmatrix} 4 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.75 \\ 1 \end{bmatrix}$$

at each iteration the vector x is multiplied by the matrix A and normalized. The subsequence x_k $(k \to \infty)$ converges to an eigenvector associated with the dominant eigenvalue.
Using the Rayleigh quotient, we can approximate the dominant eigenvalue of A:

$$\lambda = \frac{Ax \cdot x}{x \cdot x}$$

Let λ_i be the approximate dominant eigenvalue at the i-th iteration. After the first iteration:

$$\lambda_1 = \left\{ Ax_1 = \begin{bmatrix} 4 & -1 & 1 \\ 4 & -8 & 1 \\ -2 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ -0.75 \\ 1 \end{bmatrix} = \begin{bmatrix} 5.75 \\ 11 \\ 2.25 \end{bmatrix}\ and\ \ x_1 = \begin{bmatrix} 1 \\ -0.75 \\ 1 \end{bmatrix} \right\} = -0.097561$$

Continuing this process, we obtain the sequence of approximations shown in the table:

x0	**x1**	**x2**	**x3**	**x4**	**x5**	**x6**	**x7**	**x8**
1	1	0.523	-0.227	0.229	0.021	0.119	0.082	0.091
1	-0.75	1	1	1	1	1	1	1
1	1	0.205	-0.172	-0.065	-0.03	-0.102	-0.033	-0.087
λ0	**λ1**	**λ2**	**λ3**	**λ4**	**λ5**	**λ6**	**λ7**	**λ8**
1.666667	-0.097561	-3.666876	-8.05652	-6.811259	-7.979406	-7.540954	-7.724601	-7.700068

x9	**x10**	**x11**	**x12**	**x13**	**x14**	**x15**	**x16**	**x17**
0.094	0.088	0.093	0.089	0.092	0.09	0.091	0.091	0.091
1	1	1	1	1	1	1	1	1
-0.05	-0.073	-0.059	-0.068	-0.063	-0.065	-0.064	-0.065	-0.064
λ9	**λ10**	**λ11**	**λ12**	**λ13**	**λ14**	**λ15**	**λ16**	**λ17**
-7.678414	-7.71707	-7.688253	-7.711404	-7.694833	-7.705653	-7.700266	-7.700533	-7.700266

15 iterations are required to obtain successive approximations that converge when rounded to three significant digits.

Answer: $eigenvector\ \cong [0.09,\ 1, -0.06],\ \ eigenvalue\ \cong 7.70$