Newton's Law of Cooling states that the rate at which the object's temperature decreases is directly proportional to the difference between the temperature of the object and the ambient temperature. A cup of tea (temperature = 90°C) is placed in a room whose temperature is 30°C. After five minutes, the temperature of the tea has dropped to 70°C. After how much time would the temperature of the tea be 50°C?

Solution: The rate of change of the temperature dT(t)/dt, is (by Newton's Law of Cooling) proportional to the difference between the temperature of the object T(t) and the ambient temperature Ta. This means that:

$$\frac{dT(t)}{dt} = -r(T(t) - Ta) = -r \cdot \Delta T(t)$$

$$r - positive\ constant\ characteristic\ of\ the\ system$$

The solution of this differential equation:

$$\int \frac{1}{T - Ta} dT = \int -r dt$$

$$\ln(T - Ta) = -rt + C1$$

$$T(t) = Ta + e^{-rt+C1} = Ta + C \cdot e^{-rt}$$

For t=0:

$$T(0) = Ta + C \cdot e^{-r\cdot 0} \rightarrow C = T(0) - Ta = 90 - 30 = 60$$

For t=5:

$$T(5) = 70 = 30 + 60 \cdot e^{-r\cdot 5} \rightarrow r = \frac{\ln(3/2)}{5}$$

For T(t)=50:

$$50 = 30 + 60 \cdot e^{\frac{-ln\left(\frac{3}{2}\right)}{5}\cdot t} = 30 + 60 \cdot \left(\frac{2}{3}\right)^{\frac{t}{5}} \rightarrow t = \frac{-5 \cdot ln(3)}{ln(2/3)} \cong 13.548$$

Answer: $t = \frac{-5\cdot ln(3)}{ln(2/3)} \cong 13.548\, minutes$