

Question #75139, Math / Calculus

$$1) \text{ i) } \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{2hx + h^2}{h} = \lim_{h \rightarrow 0} \frac{h(2x+h)}{h} =$$

$$= \lim_{h \rightarrow 0} (2x + h) = 2x$$

$$\text{ii) } \lim_{x \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{x \rightarrow 0} \frac{x^2 + 2hx + h^2 - x^2}{h} = \lim_{x \rightarrow 0} \frac{2hx + h^2}{h} = \lim_{x \rightarrow 0} \frac{h(2x+h)}{h} =$$

$$= \lim_{x \rightarrow 0} (2x + h) = h$$

$$\text{iii) } \lim_{x \rightarrow 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} = \lim_{x \rightarrow 0} \frac{\frac{2-(2+x)}{2(2+x)}}{x} = \lim_{x \rightarrow 0} \frac{\frac{2-2-x}{4+2x}}{x} = \lim_{x \rightarrow 0} \frac{-x}{x(4+2x)} = \lim_{x \rightarrow 0} \frac{-1}{4+2x} = -\frac{1}{4}$$

$$\text{iv) } \lim_{x \rightarrow 0} \frac{(2+x)^3 - 8}{x} = \lim_{x \rightarrow 0} \frac{x^3 + 6x^2 + 12x + 8 - 8}{x} = \lim_{x \rightarrow 0} \frac{x^3 + 6x^2 + 12x}{x} = \lim_{x \rightarrow 0} \frac{x(x^2 + 6x + 12)}{x} =$$

$$= \lim_{x \rightarrow 0} (x^2 + 6x + 12) = 12$$

$$2) \text{ i) } \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}; f(x) = \frac{4}{x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\frac{4}{x + \Delta x} - \frac{4}{x}}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\frac{4x - 4(x + \Delta x)}{x(x + \Delta x)}}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{4x - 4x - 4\Delta x}{\Delta x * x(x + \Delta x)} =$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-4\Delta x}{\Delta x * x(x + \Delta x)} = \lim_{\Delta x \rightarrow 0} \frac{-4}{x^2 + x * \Delta x} = -\frac{4}{x^2}$$

$$\text{ii) } \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}; f(x) = x^2 - 4x$$

$$\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} - 4 \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x - x}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x * \Delta x + \Delta x^2 - x^2}{\Delta x} -$$

$$-4 \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{2x * \Delta x + \Delta x^2}{\Delta x} - 4 = \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} - 4 =$$

$$= \lim_{\Delta x \rightarrow 0} (2x + \Delta x) - 4 = 2x - 4 = 2(x - 2)$$

Answer provided by <https://www.AssignmentExpert.com>