Question #75004 - Math - Calculus

A transmission line with characteristic impedance 250 Ω is connected to a pure resistive load of 1000 Ω through a λ/4 line segment. Calculate the characteristic impedance of this segment to achieve perfect matching between the load and the transmission line.

Solution: input impedance of transmission line:

$$Z_{in}(l) = Z_0 \frac{Z_L + Z_0 \cdot tanh(\gamma l)}{Z_0 + Z_L \cdot tanh(\gamma l)}$$

$$Z_0 = 250 \; - \; characteristic\; impedance$$

$$Z_L = 1000 \; - \; load\; resistance$$

$$\gamma \; - \; propagation\; constant$$

$$l \; - \; length\; of\; transmission\; line$$

input impedance of lossless transmission line:

$$Z_{in}(l) = Z_0 \frac{Z_L + j \cdot Z_0 \cdot tan(\beta l)}{Z_0 + j \cdot Z_L \cdot tan(\beta l)}$$

$$\beta = \frac{2\pi}{\lambda} \; - \; wavenumber$$

for λ/4 line segment:

$$Z = \frac{Z_0^2}{Z_L} = \frac{250^2}{1000} = 62.5 \;\; \Omega$$

Answer: $Z = 62.5 \;\; \Omega$