

Answer on Question #74960 – Math / Trigonometry

Given $\tan u = -1/3$ and $\sin u < 0$ find $\sin(u/2)$. (Assume $0 \leq u < 2\pi$.)

Solution

$$\tan^2 u + 1 = \frac{1}{\cos^2 u}$$

$$\cos^2 u = \frac{1}{\tan^2 u + 1} = \frac{1}{\frac{1}{9} + 1} = \frac{9}{10}$$

$$\cos u = \frac{3}{\sqrt{10}}$$

We will use the formula:

$$\cos u = 1 - 2\sin^2 \frac{u}{2}$$

$$2\sin^2 \frac{u}{2} = 1 - \cos u$$

$$\sin^2 \frac{u}{2} = \frac{1 - \cos u}{2} = \frac{1 - \frac{3}{\sqrt{10}}}{2} = \frac{\sqrt{10} - 3}{2\sqrt{10}} = \frac{10 - 3\sqrt{10}}{20}$$

$$\sin \frac{u}{2} = \pm \sqrt{\frac{10 - 3\sqrt{10}}{20}}$$

If $\sin u < 0$ then $\sin(u/2) < 0$

So,

$$\sin \frac{u}{2} = -\sqrt{\frac{10 - 3\sqrt{10}}{20}}$$

Answer:

$$\sin \frac{u}{2} = -\sqrt{\frac{10 - 3\sqrt{10}}{20}}$$

