Answer on Question #74504 – Math – Calculus

Given:
$$f(x) = \begin{cases} 3 - kx; 1 \le x < 2\\ \frac{x^2}{4} - 3; x \ge 2 \end{cases}$$

To Find: Find the value of **"k"** so that the function is continuous at x=2by using definition.

Solution: The given function is
$$f(x) = \begin{cases} 3-kx; 1 \le x < 2\\ \frac{x^2}{4} - 3; x \ge 2 \end{cases}$$

The given function is continuous in [1, 2) and in [2, ∞), since in both interval function is a polynomial and polynomials are continuous everywhere. Now, we have to check continuity at x=2, if function is continuous at x=2 then it is continuous everywhere.

For continuity, we have

$$LHL = RHL$$

$$\therefore LHL = \lim_{x \to 2^{+}} f(x)$$

$$= \lim_{x \to 2^{+}} \left(\frac{x^{2}}{4} - 3\right)$$

$$= \left(\frac{2^{2}}{4} - 3\right) = -2$$

& RHL = $\lim_{x \to 2^{-}} f(x)$

$$= \lim_{x \to 2^{-}} (3 - kx)$$

$$= (3 - 2k)$$

Hence $(3 - 2k) = -2$

$$\Rightarrow -2k = -5$$

$$\Rightarrow k = 5/2$$

So, for k=5/2, f(x) is continuous.

If k=5/2, then f(x) is continuous everywhere on [1, ∞).