Question

If $f(x) = \frac{4x^2 - 7x - 2}{x - 2}, x \neq 2,$ find a $\delta > 0$ such that $|f(x) - 9| < \frac{1}{100}$ for $0 < |x - 2| < \delta$. Hence show that $\lim_{x \to 2} f(x) = 9$?

Solution

$$\frac{4x^2 - 7x - 2}{x - 2} = \frac{(4x + 1)(x - 2)}{x - 2} = 4x + 1$$

$$|f(x) - 9| < \frac{1}{100}$$

$$|4x + 1 - 9| < \frac{1}{100}$$

$$|4x - 8| < \frac{1}{100}$$

$$4|x - 2| < \frac{1}{100}$$

$$|x - 2| < \frac{1}{400}$$

$$\delta = \frac{1}{400}$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \left(\frac{4x^2 - 7x - 2}{x - 2} \right) = \lim_{x \to 2} \frac{(4x + 1)(x - 2)}{x - 2} = \lim_{x \to 2} (4x + 1) = 4 * 2 + 1 = 9$$

Answer: $\delta = \frac{1}{400}, \lim_{x \to 2} f(x) = 9.$